**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans: B

 We have a normal distribution = 45 and = 8.0.

Let X be the amount of time it takes to complete the repair on a customer's car.

To finish in one hour you must have X ≤ 50 so the find Pr(X > 50).

Pr(X > 50) = 1 - Pr(X ≤ 50).

Z = (X -µ )/ *σ* = (X - 45)/8.0

by using the normal table to find Pr(X ≤ 50) = Pr(Z ≤ (50 - 45)/8.0) = Pr(Z ≤ 0.625)=73.4% Probability that the service manager will not meet his demand will be = 100-73.4 = 26.6% or 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

ANS: False

Around 70% of the data falls within one standard deviation of the mean (µ+= 38+6=44)

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

ANS: True

Z=(X-µ)/ *σ*

P( X≤30)=p(Z≤(30-38)/6)=p(Z≤-1.33)= 0.0918(using z table)

Expected count=0.0918\*400= 36.72

1. If *X1*~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid*normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

ANS: 2 *X*1 will be greater scale version than *X*1 + *X*2 . If *X*1 and *X*2 are normally distributed then the sum of the random sample will be exactly same

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans.

Since we need to find out the values of a and b, which are symmetric about the mean, such that the probability of random variable taking a value between them is 0.99, we have to work out in reverse order. The Probability of getting value between a and b should be 0.99.

So the Probability of going wrong, or the Probability outside the a and b area is 0.01 (ie. 1-0.99).

The Probability towards left from a = -0.005 (ie. 0.01/2).

The Probability towards right from b = +0.005 (ie. 0.01/2).

So since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities.

By finding the Standard Normal Variable Z (Z Value), we can calculate the X values.

Z=(X- μ) / σ

For Probability 0.005 the Z Value is -2.57 (from Z Table).

Z \* σ + μ = X

Z(-0.005)\*20+100 = -(-2.57)\*20+100 = 151.4

Z(+0.005)\*20+100 = (-2.57)\*20+100 = 48.6

So, option D is correct.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

ANS: A) qnorm(0.025,45\*5,3) # 219.1201

qnorm(0.975,45\*5,3) # 230.8799

qnorm(0.025,45\*7,3) # 309.1201

qnorm(0.975,45\*7,3) # 320.8799

The Rupee Range will be [219.12, 230.87] + [309.12, 320.87] = [528.24, 551.74]

B) qnorm(0.05,45\*7,3) # 310.0654

qnorm(0.05,45\*5,3) # 220.0654

5th percentile of profit (in Rupees) = 310.0654+ 220.0654 = 530.1308

C) 2nd Division